γ-ray polarization constraints on Planck scale violations of special relativity

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Using recent polarimetric observations of the Crab Nebula in the hard X-ray band by INTEGRAL, we show that the absence of vacuum birefringence effects constrains $O(E/M)$ Lorentz violation in QED to the level $|\xi| < 9 \times 10^{-10}$ at 3σ CL, tightening by more than three orders of magnitude previous constraints. We show that planned X-ray polarimeters have the potential to probe $|\xi| \sim 10^{-16}$ by detecting polarization in active galaxies at red-shift ~ 1.

Experimental constraints on the parameters quantifying Lorentz invariance violation (LV) are of fundamental importance. Because the lowest order corrections predicted in the photon dispersion relation imply the vacuum is birefringent, observations of polarized photons from distant astronomical sources provide very promising tests. In this Letter we exploit the recently discovered linear polarization of hard X-rays from the Crab Nebula (CN) [1]. These observations show a remarkably high degree of linear polarization ($46 \pm 10\%$) and very close alignment of the polarization vector with both the optical polarization vector and the projection on the sky of the spin axis of the central neutron star. The high degree of polarization together with the lack of detectable rotation of the polarization vector of these ~ 200 keV photons whilst propagating over the intervening $\sim 6 \times 10^{21}$cm enables us to tighten existing constraints by three orders of magnitude.

Recent years have witnessed a growing interest in the possible high energy violations of local Lorentz Invariance as well as a flourishing of observational tests. Indeed, specific hints of LV arose from various approaches to Quantum Gravity [2, 3, 4, 5, 6, 7, 8, 9]. However, most tests require a well established theoretical framework to calculate reaction rates and describe the particle dynamics. Here, we work within the framework of Effective Field Theory with non-renormalizable, mass dimension 5 LV operators (see [10, 11] and references therein) restricted to QED, for which the most general dispersion relations for photons and electrons are

$$\omega_\pm^2 = k^2 \pm \xi k^3/M \qquad (1)$$
$$E_\pm^2 = p^2 + m^2 + \eta_\pm p^3/M , \qquad (2)$$

where (1) refers to photons [36] and (2) to fermions [37]. We assume M to be comparable to the Planck mass $M_{\rm Pl} \simeq 1.22 \times 10^{19}$ GeV. The constants ξ and $\eta_\pm$ indicate the strength of the LV. The $\pm$ signs denote right and left circular polarization in (1), and positive and negative helicity states of the fermion in (2). Equation (1) implies that the direction of polarization rotates during propagation due to the different velocities of the right- and left-handed circular polarizations, $v_\pm \simeq 1 \pm \xi \, k/M$. This effect is known as vacuum birefringence (VB).

Although it may seem hopeless to search directly for effects suppressed by the Planck energy scale, even tiny corrections can be magnified to measurable ones when dealing with high energies, long distances of signal propagation or peculiar reactions (see, e.g., [10, 12]). Recently $\eta_\pm$ have been constrained to have a magnitude less than 10^{-5} at 95% confidence level (CL) by a detailed analysis of the synchrotron component of the CN broadband spectrum [13], while the constraint $|\xi| \lesssim 2 \times 10^{-7}$ has been obtained by [14] considering the absence of VB effects during the propagation of optical/UV polarized light from Gamma-Ray Bursts (GRB) [38]. There are also preliminary indications, based on an analysis of the photon fraction in Ultra-High-Energy Cosmic Rays, that these coefficients might be less than 10^{-14}, though nothing conclusive can be claimed yet [15, 16].

In this work we tighten the current constraints on $O(E/M_{\rm Pl})$ suppressed LV by about three orders of magnitude for photons, by considering the limits on VB effects implied by the recently detected [1] polarized hard X-rays from the CN. Firstly, we set such constraints following the arguments by [17, 18], an approach robust against systematic uncertainties related to astrophysical modeling. We then infer tighter limits that exploit and rely on modeling of the Crab Nebula and pulsar.

Finally, we consider the constraints which future X-ray polarization measurements of extragalactic objects, e.g. Active Galactic Nuclei (AGN) will allow. This is of particular interest in the light of current experimental efforts to build X-ray polarimeters [19, 20, 21, 22].

During propagation over a distance d [39], the polarization vector of a linearly polarized plane wave with mo-

mentum k rotates through an angle [4, 17, 18, 23],

$$\theta(k,d) = \frac{\omega_+(k) - \omega_-(k)}{2}d \simeq \xi \frac{k^2 d}{2\,M_{\mathrm{Pl}}} \ . \qquad (3)$$

Observations of polarized light from a distant source can constrain $|\xi|$ in two ways, depending on the amount of available information on both the observational and the theoretical (i.e. source modeling) side:

1. Since detectors have a finite energy bandwidth, eq. (3) is never probed in real situations. Rather, if some net amount of polarization is measured in the band $k_1 < E < k_2$, an order-of-magnitude constraint arises from the fact that if the angle of polarization rotation (3) were to differ by more than $\pi/2$ over this band, the detected polarization would fluctuate sufficiently for the net signal polarization to be suppressed [17, 18]. From (3), this constraint is

$$\xi \lesssim \frac{\pi\,M_{\mathrm{Pl}}}{(k_2^2 - k_1^2)d(z)} \ , \qquad (4)$$

   This just requires that any intrinsic polarization (at source) is not completely washed out during signal propagation. It thus relies on the mere detection of a polarized signal, without considering the observed polarization degree. A more refined limit can be obtained by calculating the maximum observable polarization degree, given the maximum intrinsic value [24]:

$$\Pi(\xi) = \Pi(0)\sqrt{\langle\cos(2\theta)\rangle_{\mathcal{P}}^2 + \langle\sin(2\theta)\rangle_{\mathcal{P}}^2}, \qquad (5)$$

   where $\Pi(0)$ is the maximum intrinsic degree of polarization, θ is defined in eq. (3) and the average is weighted over the source spectrum and instrumental efficiency, represented by the normalized weight function $\mathcal{P}(k)$ [17]. Conservatively, one can set $\Pi(0) = 100\%$, but a lower value can sometimes be justified on the basis of source modeling. Using (5), one can then cast a constraint by requiring $\Pi(\xi)$ to exceed the observed value.

2. Suppose that polarized light measured in a certain energy band has a position angle θ_{obs} with respect to a fixed direction. At fixed energy, the polarization vector rotates by the angle (3) [40]; if the position angle is measured by averaging over a certain energy range, the final net rotation $\langle\Delta\theta\rangle$ is given by the superposition of the polarization vectors of all the photons in that range:

$$\tan(2\langle\Delta\theta\rangle) = \frac{\langle\sin(2\theta)\rangle_{\mathcal{P}}}{\langle\cos(2\theta)\rangle_{\mathcal{P}}} \ , \qquad (6)$$

   where θ is given by (3). If the position angle at emission θ_{i} in the same energy band is known from
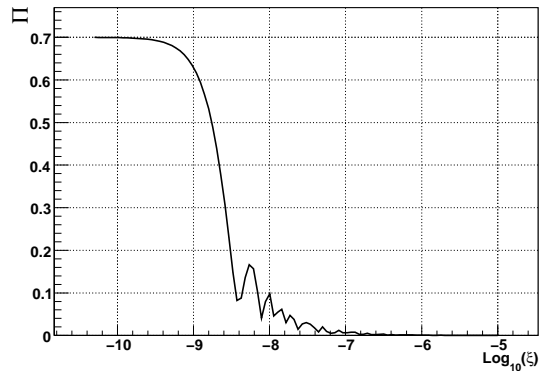

FIG. 1: Constraint for the polarization degree. Dependence of Π on ξ for the distance of the CN and photons in the 150–300 keV range, for a constant $\mathcal{P}(k)$.

a model of the emitting source, a constraint can be set by imposing

$$\tan(2\langle\Delta\theta\rangle) < \tan(2\theta_{\mathrm{obs}} - 2\theta_{\mathrm{i}}) \ . \qquad (7)$$

Although this limit is tighter than that obtained from the previous methods, it clearly hinges on assumptions about the nature of the source, which may introduce significant uncertainties.

In the case of the Crab Nebula, a $(46 \pm 10)\%$ degree of linear polarization in the 100 keV − 1 MeV band has recently been measured by the INTEGRAL mission [1, 25]. This measurement uses all photons within the SPI instrument energy band. However the convolution of the instrumental sensitivity to polarization with the detected number counts as a function of energy, $\mathcal{P}(k)$, is maximized and approximately constant within a narrower energy band (150 to 300 keV) and falls steeply outside this range [26]. For this reason we shall, conservatively, assume that most polarized photons are concentrated in this band. Given $d_{\mathrm{Crab}} = 1.9$ kpc, $k_2 = 300$ keV and $k_1 = 150$ keV, eq. (4) leads to the order-of-magnitude estimate $|\xi| \lesssim 2 \times 10^{-9}$. A more accurate limit follows from (5). In the case of the CN there is a robust understanding that photons in the range of interest are produced via the synchrotron proces, for which the maximum degree of intrinsic linear polarization is about 70% (see e.g. [27]). Figure 1 illustrates the dependence of Π on ξ for the distance of the CN and for $\Pi(0) = 70\%$. The requirement $\Pi(\xi) > 16\%$ (taking account of a 3σ offset from the best fit value 46%) leads to the constraint (at 99% CL)

$$|\xi| \lesssim 6 \times 10^{-9} \ . \qquad (8)$$

It is interesting to notice that X-ray polarization measurements of the CN already available in 1978 [28], set a constraint $|\xi| \lesssim 5.4 \times 10^{-6}$, only one order of magnitude less stringent than that reported in [14].

Constraint (8) can be tightened by exploiting the current astrophysical understanding of the source. The CN is a cloud of relativistic particles and fields powered by a rapidly rotating, strongly magnetized neutron star. Both the *Hubble Space Telescope* and the *Chandra* X-ray satellite have imaged the system, revealing a jet and torus that clearly identify the neutron star rotation axis [29]. The projection of this axis on the sky lies at a position angle of $124.0° \pm 0.1°$ (measured from North in anti-clockwise). The neutron star itself emits pulsed radiation at its rotation frequency of 30 Hz. In the optical band these pulses are superimposed on a fainter steady component with a linear polarization degree of 30% and direction precisely aligned with that of the rotation axis [30]. The direction of polarization measured by INTEGRAL-SPI in the γ-rays is $\theta_{obs} = 123° \pm 11°$ (1σ error) from the North, thus also closely aligned with the jet direction and remarkably consistent with the optical observations.

This compelling (theoretical and observational) evidence allows us to use eq. (7). Conservatively assuming $\theta_i - \theta_{obs} = 33°$ (i.e. 3σ from θ_i, 99% CL), this translates into the limit

$$|\xi| \lesssim 9 \times 10^{-10} \,, \qquad (9)$$

and $|\xi| \lesssim 6 \times 10^{-10}$ for a 2σ deviation (95% CL). Figure 2 shows $\tan(2\theta_f)$ as function of ξ. The left–hand panel reports the global dependence (the spikes correspond to rotations by $\pi/4$), while the right–hand panel focuses on the interesting range of values [41].

The constraints presented in (8) and (9) are remarkably strong. Although based on a cumulative effect, they are achieved using a local (Galactic) object. The reason lies, on the one hand, in the quadratic dependence of θ on the photon energy, in contrast with the linear gain given by distance (see e.g. eq. (3)). On the other hand, the robust theoretical understanding of the CN has enabled us to strengthen the constraints significantly.

Further improvements on LV constraints via birefringenge are expected thanks to the forthcoming high-energy polarimeters, such as XEUS [31], PoGoLite [22], Polar-X [20] and Gamma Ray Imager [32] which will provide an unprecedented sensitivity, sufficient to detect polarized light at a few % levels also in extragalactic sources. The LV limits will be optimized by balancing between source distance and observational energy range depending on the detector sensitivity. This is illustrated in Fig. 3, where the strength of the possible constraints (cast with the first, most general method described above) is plotted versus the distance of sources (in red-shift z) and for different energy bands (medium X- and γ-rays). Remarkably, constraints of order $|\xi| < O(10^{-16})$ could be placed if some polarized distant sources ($z \sim 1$) will be observed by such instruments at 1 MeV.

[1] Dean, A. J. et al., Science **321**,1183 (2008).
[2] V. A. Kostelecky and S. Samuel, Phys. Rev. **D39**, 683 (1989).
[3] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos, and S. Sarkar, Nature **393**, 763 (1998), astro-ph/9712103.
[4] R. Gambini and J. Pullin, Phys. Rev. **D59**, 124021 (1999), gr-qc/9809038.
[5] S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane, and T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001), hep-th/0105082.
[6] J. Lukierski, H. Ruegg, and W. J. Zakrzewski, Ann. Phys. **243**, 90 (1995), hep-th/9312153.
[7] G. Amelino-Camelia and S. Majid, Int. J. Mod. Phys. **A15**, 4301 (2000), hep-th/9907110.
[8] C. P. Burgess, J. Cline, E. Filotas, J. Matias, and G. D. Moore, JHEP **03**, 043 (2002), hep-ph/0201082.
[9] C. Barcelo, S. Liberati, and M. Visser, Living Rev. Rel. **8**, 12 (2005), gr-qc/0505065.
[10] D. Mattingly, Living Rev. Rel. **8**, 5 (2005), gr-qc/0502097.
[11] R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003), hep-ph/0301124.
[12] G. Amelino-Camelia, New J. Phys. **6**, 188 (2004), gr-qc/0212002.
[13] L. Maccione, S. Liberati, A. Celotti, and J. Kirk, Journal of Cosmology and Astroparticle Physics **2007**, 013 (2007), arXiv:0707.2673 [astro-ph].
[14] Y.-Z. Fan, D.-M. Wei, and D. Xu, Mon. Not. Roy. Astron. Soc. **376**, 1857 (2006), astro-ph/0702006.
[15] M. Galaverni and G. Sigl, Phys. Rev. Lett. **100** (2007), 0708.1737.
[16] L. Maccione and S. Liberati (2008), 0805.2548.
[17] R. J. Gleiser and C. N. Kozameh, Phys. Rev. **D64**, 083007 (2001), gr-qc/0102093.
[18] T. A. Jacobson, S. Liberati, D. Mattingly, and F. W. Stecker, Phys. Rev. Lett. **93**, 021101 (2004), astro-ph/0309681.
[19] E. Costa et al. (2006), astro-ph/0603399.
[20] E. Costa et al. (2006), astro-ph/0609576.
[21] F. Muleri et al. (2006), astro-ph/0609573.
[22] Y. Kanai, M. Ueno, J. Kataoka, M. Arimoto, N. Kawai, K. Yamamoto, T. Mizuno, Y. Fukazawa, M. Kiss, T. Ylinen, et al., Nuclear Instruments and Methods in Physics Research A **570**, 61 (2007).
[23] T. Jacobson, S. Liberati, and D. Mattingly, Annals Phys. **321**, 150 (2006), astro-ph/0505267.
[24] W. H. McMaster, Rev. Mod. Phys. **33**, 8 (1961).
[25] A. N. Parmar, C. Winkler, P. Barr, L. Hansson, E. Kuulkers, R. Much, and A. Orr, in *X-Ray and Gamma-Ray Telescopes and Instruments for Astronomy. Edited by Joachim E. Truemper, Harvey D. Tananbaum. Proceedings of the SPIE, Volume 4851, pp. 1104-1112* (2003).
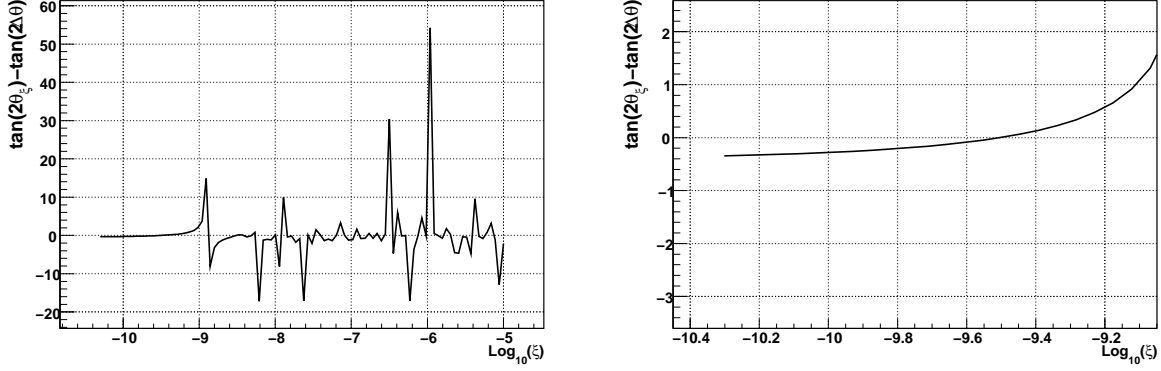
FIG. 2: Constraint for the polarization rotation case. Left panel: dependence of $\tan(2\theta_f)$ on ξ. The spikes correspond to rotations by $\pi/4$. Right panel: a zoom-in on the interesting range of values. The constraint is cast according to eq. (6).
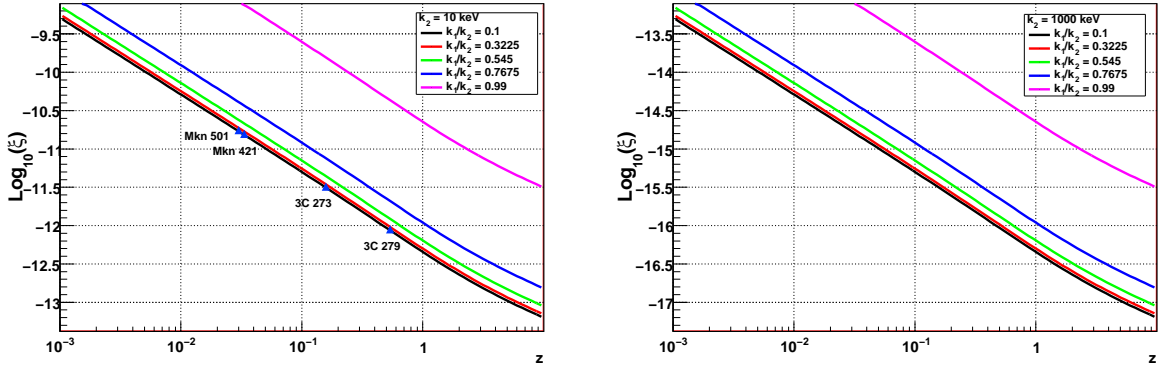

FIG. 3: Expected constraints from medium X- and soft γ-ray polarimetry of extra-galactic sources. High energy scale $k_2 = 10$ keV (left panel) and 1 MeV (right panel), with $\kappa \equiv k_1/k_2$ from 0.1 to 0.99. Points in the left panel refer to the characteristics of a new generation X-ray polarimeter [31] assuming that polarization is detected from the mentioned objects. The constraints are derived as in case of eq. (4) for a concordance cosmology ($\Omega_m = 0.28$, $\Omega_\Lambda = 0.72$ and $H_0 = 73$ km s^{-1}Mpc^{-1}).

[26] S. McGlynn et al. (2007), astro-ph/0702738.
[27] J. Petri and J. G. Kirk, Astrophys. J. **627**, L37 (2005), astro-ph/0505427.
[28] M. C. Weisskopf, E. H. Silver, H. L. Kestenbaum, K. S. Long, and R. Novick, ApJL **220**, L117 (1978).
[29] C. Y. Ng and R. W. Romani, Astrophys. J. **601**, 479 (2004), astro-ph/0310155.
[30] G. Kanbach, A. Slowikowska, S. Kellner, and H. Steinle, AIP Conf. Proc. **801**, 306 (2005), astro-ph/0511636.
[31] E. Costa et al. (2002), astro-ph/0207440.
[32] J. Knodlseder et al. (2007), 0707.4627.
[33] I. G. Mitrofanov, Nature **426**, 139 (2003).
[34] W. Coburn and S. E. Boggs, Nature **423**, 415 (2003), arXiv:astro-ph/0305377.
[35] R. E. Rutledge and D. B. Fox, Mon.Not.R.Astron.Soc. **350**, 1288 (2004), arXiv:astro-ph/0310385.
[36] This kind of dispersion relation was also derived in some semi-classical limit of Loop quantum gravity [4].
[37] For positrons we have $\eta_\pm^{\text{pos}} = -\eta_\mp^{\text{el}}$ [23].

[38] Even stronger constraints, $O(10^{-14})$, were claimed in [18, 33] from GRB 021206 observations [34]; however the result was later contested [35].
[39] For an extragalactic object at redshift z, the (cosmological) distance is given by $d(z) = \frac{1}{H_0}\int_0^z \frac{1+z'}{\sqrt{\Omega_\Lambda + \Omega_m(1+z')^3}}\,dz'$, which includes a $(1+z')^2$ factor in the integrand to take into account the red-shift acting on the photon energies. As usual, H_0 is the present value of the Hubble parameter, while Ω_Λ and Ω_m represent the density fractions of cosmological constant and matter in the Universe, respectively.
[40] Faraday rotation is negligible at such energies.
[41] Note that the constraint (8) rules out the possibility that the polarization angle is close to the expected one after rotating by some multiple of π (the polarization angle is defined on the interval $[0, \pi]$).